**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

> mu<-45

> sigma<-8

> pnorm(50,45,8) #probability that the service manager can meet his commitment#

[1] 0.7340145

> 1-pnorm(50,45,8) #probability that the service manager cannot meet his commitment#

[1] 0.2659855

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: False

> pnorm(44,38,6,lower.tail = FALSE)

[1] 0.1586553

Probability that the employee will be greater than age of 44 = 15.86%

> pnorm(44,38,6)-pnorm(38,38,6)

[1] 0.3413447

Probability of number of employees between 38-44 years of age =34.13%

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True

> Expected\_employees\_under30 = 400\*pnorm(30,38,6)

> Expected\_employees\_under30

[1] 36.48449

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

*X1* ~ *N*(μ, σ2)

*X*2 ~ *N*(μ, σ2)

2 *X*1~ *N*(2μ, 2σ2)

This means that 2 *X*1 independent normally distributed random variables is normal, with its mean being 2μ, and its variance being 2σ2

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X1+X2 = N(μx1+ μx2, σx12+ σx22 )

This means that the sum of two independent normally distributed random variables is normal, with its mean being the sum of the two means, and its variance being the sum of the two variances

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D: 48.5, 151.5

Mu=100

Sigma=20

Z=(x-mu)/sigma

X=z\*sigma+mu

a=+Z\*sigma+mu = -2.575829\*20+100= 48.48341

b=- Z\*sigma+mu =-(-2.575829\*20+100)= 151.5166

> Mu=100

> sigma=20

> #confideance\_interval=99%

> e=1-0.99

> qnorm(e/2)

[1] -2.575829

> a=qnorm(e/2)\*sigma+Mu

> a

[1] 48.48341

> b=-qnorm(e/2)\*sigma+Mu

> b

[1] 151.5166

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans:

Profit1 ~ N(5, 32)

Mu1=5

Sigma1=3

Profit2 ~ N(7, 42)

Mu2=7

Sigma2=4

Assume that $1 = Rs. 45

Combined Profit

Profit3 ~ N(5+7, 32 +42) = N(12,25)=N(12, 52)

Mu=12

Sigma=5

> Mu=12

> sigma=5

> #confideance\_interval=95%

> e=1-0.95

> qnorm(e/2)

[1] -1.959964

> a=qnorm(e/2)\*sigma+Mu

> b=-qnorm(e/2)\*sigma+Mu

> a

[1] 2.20018

> b

[1] 21.79982

> Lower\_Limit=a\*45

> Upper\_Limit=b\*45

> Lower\_Limit

[1] 99.0081

> Upper\_Limit

[1] 980.9919

1. Specify the 5th percentile of profit (in Rupees) for the company

> Mu=12

> sigma=5

> #confideance\_interval=90%

> e=1-0.90

> qnorm(e/2)

[1] -1.644854

> FifthPercentile\_of\_Profit\_USD=qnorm(e/2)\*sigma+Mu

> FifthPercentile\_of\_Profit\_INR=FifthPercentile\_of\_Profit\_USD\*45

> FifthPercentile\_of\_Profit\_INR

[1] 169.9079

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Profit1 ~ N(5, 32) has a larger probability of making a loss in a given year as compared to Profit2 ~ N(7, 42).

As Profit1 has lower profit mean as compared to Profit2. Also Profit2 has higher standard deviation that means profit range of second division is more spread out or dispersed as compared to division one.